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A numerical experiment to determine the soil water contents in the unsaturated zone and the water table response under transient ponding conditions

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Abstract

A model based on analytical solutions derived from the 2-phase flow theory for infiltration and drainage processes under transient supply conditions is presented here. The mathematical derivations are based on the knowledge of unsaturated flow in porous media, on the continuity equation and Darcy equation. The main assumption concerns the rectangular shape of the water content profile with depth which leads to an abrupt change of the water contents as the front goes by. A sequence of infiltration and drainage fronts are analysed and the results are presented. New is the approach of combining the saturated and the unsaturated zones in one efficient methodology using analytical solutions.

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1. Introduction

In soil sciences, hydrology and agricultural sciences, water content plays an important role regarding groundwater recharge, agriculture, soil chemistry and ecology. The interest in modeling soil water dynamics in the unsaturated zone has been increased in the last 30 years [1] [2] [3].

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Scientific efforts have aimed towards a predictive approach for the soil water distribution over space and time for different purposes such as a better assessment of fire risk or a more comprehensive understanding of the seasonal dynamics of the water dependent terrestrial ecosystems or an accurate simulation of the groundwater recharge process. Laboratory experiments have given insight into the front morphology [4]. Mathematical models have given evidence of the accuracy of the current approaches for mildly unsaturated flow in the vadose zone [5] and for saturated aquifer recharge [6]. Techniques for field estimation of the water content have been improved and their accuracy tested [7].

The purpose of this study is to describe the evolution in time of the soil water content and to offer a tool to interpret the field values of water contents under transient water supply conditions at the surface. The approach is based on soil water balance, averaged values of the water content below and behind the fronts which are assumed to be rectangular shaped. New is the approach of combining the saturated and unsaturated zones in one efficient methodology using analytical solutions.

2. Methodology

A methodology previously applied for the interpretation of the water table levels at an artificial groundwater recharge plant in Fresno, California [8] has been used for determining the soil water contents and the aquifer response. Based on the theory of multi-phase flow and the current understanding of unsaturated flow in porous media, approximate equations for the evolution of water content with time were derived. The basic approximation consists of assimilating the water content profile to a rectangle. The water content is assumed to be uniform from the surface to the descending front downstream which propagates further into the assumed initially uniform water content. This approximation for the rectangular profile is especially valid down into the column once the wetting front has passed it and previous studies have shown that this approximation is realistic [9]. The water content in the upper rectangular zone (see Fig.1) is calculated both with a “mass balance” approach and with a “dynamic” approach. The first one is based on conservation of mass and the second one is based on the two-phase flow theory and on the definition of total velocity. Given that at early times capillary dominates, the dynamic estimate is used when the front first starts to proceed down. Beyond that time a weighted average of the two estimates is used.

3. Mathematical derivations

3.1. Infiltration phase

Let i_{sr} denote the infiltration rate associated with the supply rate. The water content between the surface and the descending front is θ . Below the descending front and above the water table the water content has a uniform value denoted θ_{rech} (the subscript referring to recharge). The flow associated with that water content is the recharge rate to the water table. At the beginning of the simulation the water content is assumed to be uniform. Let z_f denote the depth of a descending front and z_{rf} denote the height of the current water table (Fig. 1). D_{wt} is the depth to the initial position of the water table and $\tilde{\theta}$ is the water content at natural saturation. Let q_{rech} designate the recharge rate into the current water table. The cumulative net infiltration water depth since time zero must fill the available pore space under the rectangular water content profiles, thus the relation:

$$W = \int_0^t \{i_{sr}(\tau) - q_{rech}(\tau)\} d\tau = z_f(\theta - \theta_i) + (\theta_{rech} - \theta_i)(D_{WT} - z_f - z_{rf}) + (\tilde{\theta} - \theta_i)z_{rf} \quad (1)$$

Note that in the early phases of infiltration into the soil the initial water content is immobile and no recharge is taking place. In this case the right hand side of Eq.(1) reduces to its first term since $\theta_{rech} = \theta_i$ and $z_{rf} = 0$. In Figure 1 the descending front is not the first one; otherwise the value of θ_{rech} would have been θ_i . The descending front in the figure is a wetting one. It could have been a draining one and then θ would have been less than θ_{rech} . Once a descending front hits the reflected front the two fronts merge and the value of θ_{rech} changes to the value of the water content of the descending front as it hits the water table. A new descending front starts immediately from the surface and from that time on one keeps track of the water depth under that descending front.

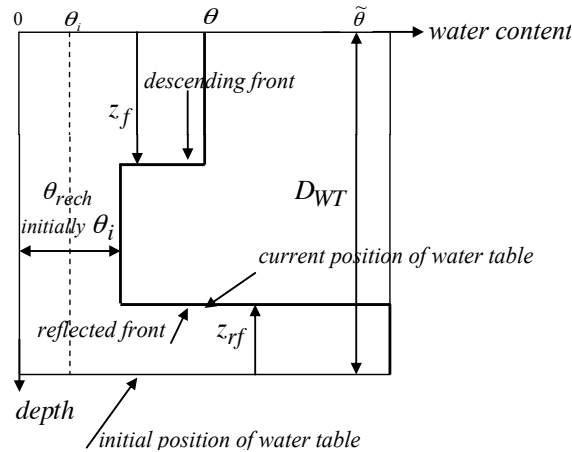


Fig. 1. Water content profile

3.2. Determination of the position of the descending front

The water depth at the end of the time step is:

$$W = (\theta - \theta_{rech})z_f \quad (2)$$

If the front is a draining one the value of θ will be less than θ_{rech} and the value of W will be negative.

3.3. Mass balance estimate of the water content

Conservation of mass will require that the increment of water depth in the profile must be the difference between what enters the soil at the surface and what leaves at the descending front. What enters is i_{sr} and what leaves, having assumed that the flow leaving the descending front is driven by gravity alone, is $q_{rech} = \tilde{K}(\theta_{rech}^*)^p$. \tilde{K} is the hydraulic conductivity of the soil at natural saturation, the star over a water content indicates that it has been normalized ($\theta^* = \frac{\theta - \theta_{res}}{\tilde{\theta} - \theta_{res}}$) and p is the exponent for the power expression of relative permeability. θ_{res} is the residual water content. One can write then:

$$\frac{dW}{dt} = i_{sr} - q_{rech} = (\theta - \theta_{rech}) \frac{dz_f}{dt} + \frac{d(\theta - \theta_{rech})}{dt} z_f \quad (3)$$

The velocity of propagation of the descending front [10] is:

$$\frac{dz_f}{dt} = \left\{ \frac{\tilde{K}(\theta^*)^p - q_{rech}}{\theta - \theta_{rech}} \right\} \quad (4)$$

Using Eqs.(4) and (2) to eliminate z_f in Eq.(3) leads to an equation uniquely in terms of θ^* , which can be rewritten using the relative permeability k_{rw} as the dependent variable. The final equation after separating the variables takes the form:

$$\frac{dk_{rw}}{k_{rw}(i_{sr}^* - k_{rw})} = \frac{p\tilde{K}[1 - \frac{\theta_{rech}^*}{\theta^*}]}{[W^0 + (i_{sr} - q_{rech})(t - t^0)]} dt \quad (5)$$

where the normalized infiltration rate has been defined: $i_{sr}^* = \frac{i_{sr}}{\tilde{K}}$

If the infiltration rate is not zero one obtains:

$$k_{rw} = \left\{ \frac{E}{E - 1 + \frac{i_{sr}^*}{k_{rw}^0}} \right\} i_{sr}^* \quad (6)$$

where:

$$E = e^{(1 - \frac{\theta_{rech}^*}{\theta_{est}^*}) \frac{pi_{sr}}{(i_{sr} - q_{rech})} \ln \left[1 + \frac{i_{sr} - q_{rech}}{W^0} \right]} \quad (7)$$

Once k_{rw} obtained the value of θ^* follows as:

$$\theta^* = (k_{rw})^{1/p} \quad (8)$$

Then the value of z_f can be obtained from Eq.(2).

3.4 Determination of the current position of the water table (reflected front)

The recharge into the water table leads to a rise of the water table and to a lateral discharge (transferred laterally to the aquifer away from the recharge zone). The velocity of the reflected front is:

$$\frac{dz_{rf}}{dt} = \frac{q_{rech} - q_{lateral}}{(\hat{\theta} - \theta_{rech})} \quad (9)$$

where $q_{lateral}$ is the lateral discharge velocity (dimension of length per time). The value of that discharge depends upon the head difference of the water table in the recharge zone and the head at some distance away from the recharge zone where the flow in the aquifer satisfies the Dupuit-Forchheimer

assumption [11]. The drive for the flow to recharge the aquifer comes from the saturated depth in the soil and the resistance to that flow comes from the difficulty of that discharge to turn into the aquifer away from the recharge zone. Previous work [6] has shown that this lateral resistance can be calculated under the assumption of full penetration but corrected by a “turning” factor “leading to the expression:

$$q_{lateral} = \frac{\lambda}{1 + \lambda z_{rf} / \tilde{K}} (z_{rf} - h_{far}) = \kappa (z_{rf} - h_{far}) \quad (10)$$

having defined

$$\lambda = K_H \frac{e}{B \Delta x_{far}} \tau_f \quad (11)$$

and

$$\kappa = \frac{\lambda}{1 + \lambda z_{rf} / \tilde{K}} \quad (12)$$

where K_H is the horizontal hydraulic conductivity of the aquifer, e is the saturated thickness of the aquifer below the recharge area, B is half the width of the recharge area, τ_f (the turning factor) is a dimensionless factor accounting for the difficulty of the vertical recharge to turn horizontally into the aquifer, and h_{far} is the head, with datum at the initial water table position, at a distance Δx_{far} of value safely taken at two aquifer thicknesses away from the edge of the recharge area. The propagation rate of the reflected front is:

$$(\tilde{\theta} - \theta_{rech}) \frac{dz_{rf}}{dt} = q_{rech} - \kappa (z_{rf} - h_{far}) \quad (13)$$

The solution of this differential equation gives the position of the water table z_{rf} .

3.5 Determination of the infiltration rate

If the supply rate is greater than the infiltration rate, it is still possible for the entire supply rate to infiltrate due to the action of capillarity, provided that h is less than h_{ce} where h is determined as

$$h = \frac{(sr^* - 1)W}{(\tilde{\theta} - \theta_{res})(1 - \theta_{rech}^*)} - [1 - (\theta_{rech}^*)^{p-M}] H_{cS} \leq h_{ce} \quad (14)$$

In that case saturation occurs at the soil surface but not ponding. If on the other hand h exceeds h_{ce} then ponding occurs. The supply rate [12] does not infiltrate in totality but only at the capacity infiltration rate which at that moment, and given the water content profile, is:

$$i_{sr} = \frac{\tilde{K} \{ (\tilde{\theta} - \theta_{res})(1 - \theta_{rech}^*) [H_{cM} - H_{cS} (\theta_{rech}^*)^{p-M}] \}}{W} + \tilde{K} \quad (15)$$

4. Numerical Experiment

The residual water content is $\theta_r = 0.20$, the water content at natural saturation is $\tilde{\theta} = 0.40$, the initial water content is $\theta_i = 0.20$ and the entry pressure head is 20 cm. The exponent for the relative permeability power law (Eq.11) is $p=6.0$ and that M for the capillary pressure head ($h_c = h_{ce}(\theta^*)^{-M}$) is 2.0. The hydraulic conductivity of the soil at natural saturation is $\tilde{K} = 20$ cm/hour, the aquifer horizontal conductivity is 65 cm/hour, the turning factor is 1.0 and the initial depth to the water table is 3.5 meters. The half width of the recharge zone is 2.0 meters. The aquifer thickness is 20.0 meters. The distance Δx_{far} is assumed twice the aquifer thickness. (Additional numerical simulations and comparison with field tests are available [13]).

Figure 2 shows the evolution with time of the fronts and of the water table. One can see from the figure that the descending front hits the water table and the 2 fronts merge at times 3.25 hours. One can recognize these times graphically when a black solid dot coincides with an open red square. Immediately as a descending front merges with the water table, another front initiates at the surface. Between times 3.25 and 3.75, 7.25 and 8.75 the soil is entirely saturated (waterlogging, when water table rises to the soil surface). Note that in this case the supply rates exceed the conductivity at natural saturation (20 cm/hour or 1.67 cm/period). The infiltration rate then can only be as high as the lateral recharge rate. This is well shown in Figure 4 too. Some of the water supply will pond on the surface and will be available for later infiltration. In these simulations the ponded depth was not considered to increase the infiltration capacity of the soil at that moment.

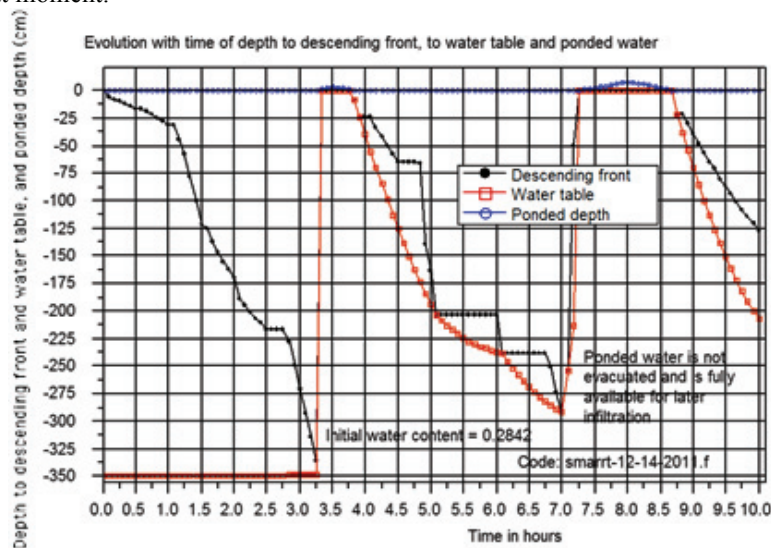


Fig. 2. Position of descending fronts and of the water table

Figure 3 shows the water contents behind the descending fronts and just above the water table. As soon as a front hits the water table the water content above the water table takes the value of that of the descending front. This is very clear on the figure. One recognizes a drainage front when the black line is below the red line. Figure 4a shows the cumulative infiltration depth, the net cumulative infiltration

(infiltration minus lateral recharge) and the cumulative water depth within each descending front. Finally, Figure 4b shows the supply rate, the infiltration rate and the recharge rate to the aquifer.

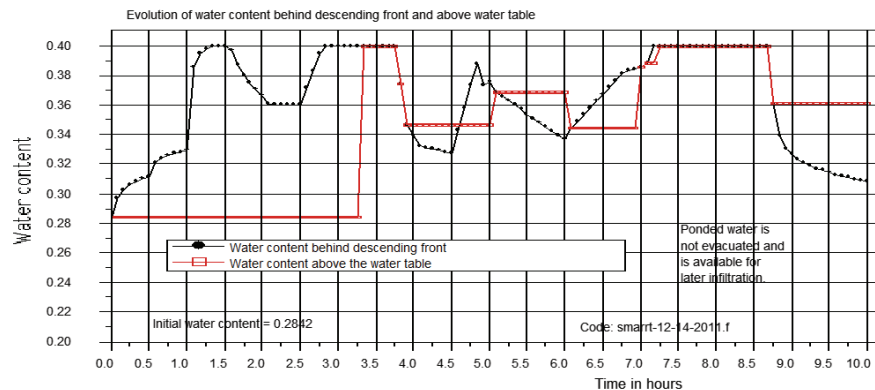
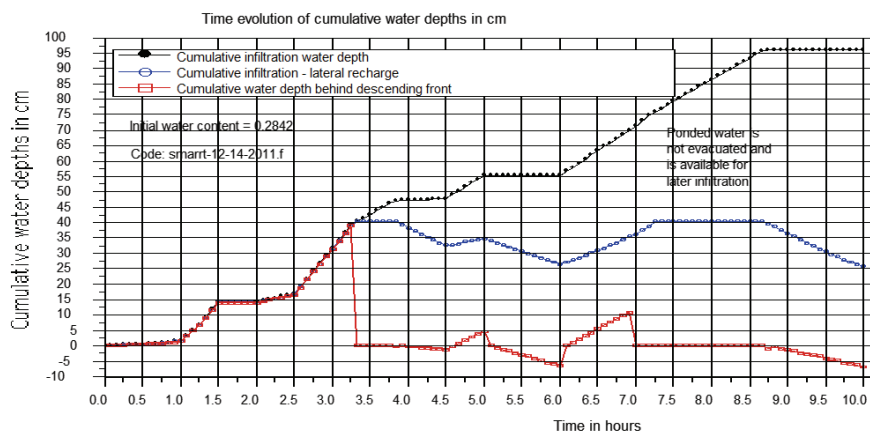
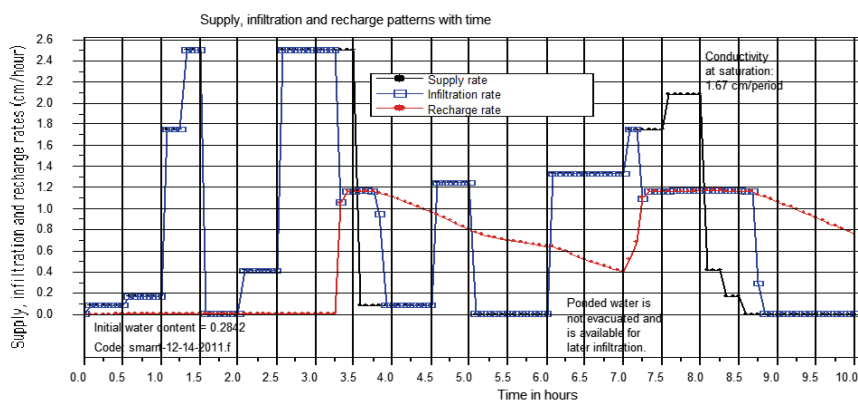


Fig. 3. Evolution of the water contents with time.



a)



b)

Fig. 4. (a) cumulative water depth with time; (b) supply and infiltration rates with time.

5. Conclusions

A computational model and the first numerical results are presented here. Although the accuracy of the results, in term of water content, has not been yet completely verified, the accuracy of the overall approach has already been previously tested. Thus the model can be used as a tool in predicting the time at which recharge occurs and at which rate. It enables also to determine the position of the front at a prescribed time and the water content at a prescribed depth. This knowledge is a prerequisite for selecting the time interval for field measurements before the monitoring phase starts and evaluate in a pre-monitoring phase the impact of different hydrological scenarios.

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